*sheet 1 (1)

[1] using partial fraction tequnie find $e(0)$ and $e(10)$, then check the value of $e(0)$ using the initial value theorem for

$$E(Z) = \frac{1}{(Z-1)(Z-0.3)}$$

$$E(Z) = \frac{AZZ^{-1}}{Z-1} + \frac{BZZ^{-1}}{Z-0.3} \longrightarrow e(K) = A\,u(K-1) + B(0.3)^{K-1}\,u(K-1)$$

$$e(K) = u(K-1)\left[\frac{10}{7} - \frac{10}{7}(0.3)^{K-1}\right]$$

$$\therefore e(0) = \underbrace{u(-1)}_{0}\left[\frac{10}{7} - \frac{10}{7}(0.3)^{-1}\right] = 0$$

$$\therefore e(10) = \underbrace{u(9)}_{1}\left[\frac{10}{7} - \frac{10}{7}(0.3)^{9}\right] = 1.428$$

$$\therefore e(0) = \lim_{Z\to\infty} E(Z) = \lim_{Z\to\infty} \frac{1}{(Z-1)(Z-0.3)} = 0$$

[2] A function $e(t) = A\cos(wt)$ is sampled every $T = 0.2$ sec. If the Z-transform of the resultant number sequence, find A and w for:

$$E(Z) = \frac{3Z(Z-0.6967)}{Z^2 - 1.3934Z + 1}$$

$$\cos(wt) = \frac{Z(Z - \cos wT)}{Z^2 - 2Z\cos wT + 1} \qquad \therefore A = 3$$

$$\cos(wT) = 0.6967 \longrightarrow wT = [\cos^{-1} 0.6967] \times \frac{\pi}{180} = 0.8$$

$$w = \frac{0.8}{0.2} = 4 \text{ rad/sec}$$

[3] Solve the given difference equation for $y(K)$ using the sequential technique and the Z-transform for

$$y(K+2) - \frac{3}{4}y(K+1) + \frac{1}{8}y(K) = e(K) \quad , y(0) = y(1) = 0 \quad e(K) = 1,\ K = 0, 1, 2, \ldots$$

$$Z^2 Y(Z) - \underbrace{Z^2 y(0)}_{0} - \underbrace{Z y(1)}_{0} - \frac{3}{4} Z Y(Z) + \underbrace{\frac{3}{4} Z y(0)}_{0} + \frac{1}{8} Y(Z) = E(Z)$$

$$Y(Z)\left[Z^2 - \frac{3}{4}Z + \frac{1}{8}\right] = \frac{Z}{Z-1}$$

$$\therefore Y(Z) = Z\,\frac{1}{(Z-1)(Z-0.5)(Z-0.25)} = \frac{ZA}{(Z-1)} + \frac{ZB}{Z-0.5} + \frac{ZC}{Z-0.25}$$

$$Y(Z) = \frac{8/3\,Z}{Z-1} - \frac{8Z}{Z-0.5} + \frac{16/3\,Z}{Z-0.25}$$

$$y(K) = 8/3\,u(K) - 8(0.5)^K u(K) + \frac{16}{3}(0.25)^K u(K)$$

$$y(\infty) = \lim_{Z \to 1} (Z-1) F(Z) = \lim_{Z \to 1} (Z-1) \frac{Z}{(Z-1)(Z-0.5)(Z-0.25)} = \frac{1}{(0.5)(0.75)} = \frac{8}{3}$$

4 A function $E(t)$ sampled, and the resultant o/p sequence has the following Z transform, find Z.T of $e(t-3T)\,u(t-3T)$

, find Z.T of $e(t+T)\,u(t+T)$

$$E(Z) = \frac{Z^3}{Z^3+3Z^2+5Z-9}$$

$$e(t-3T).u(t-3T) = \frac{Z^{-3}\ Z^3}{Z^3+3Z^2+5Z-9}$$

$$Z[e(t+T).u(t+T)] = \frac{Z^4}{Z^3-3Z^2+5Z-9} - Z\,e(0)$$

$$e(0) = \lim_{Z \to \infty} \frac{1}{1-\frac{3}{Z}+\frac{9}{Z^2}-\frac{5}{Z^3}} = 1$$

5 Given a discrete Time dynamic system represented by the difference equation solve for $X(K)$ as a function of Time K

$$X(K+2) + 3X(K+1) + 2X(K) = e(K)$$

$$e(K) = \begin{cases} 1 & , K=0 \\ 0 & , \text{otherwise} \end{cases} \qquad \text{withe } X(0)=0 \ , \ X(1)=-1$$

$$Z^2 X(Z) - Z^2 \overset{0}{X(0)} - Z\overset{-1}{X(1)} + 3Z\,X(Z) - 3Z X(0) + 2X(Z) = E(Z)$$

$$Z^2 X(Z) + Z + 3Z\,X(Z) + 2X(Z) = 1$$

$$X(Z) = \frac{1-Z}{Z^2+3Z+2} = \frac{A}{Z+1} + \frac{B}{Z+2} = \frac{2\,Z\,Z^{-1}}{Z-(-1)} - \frac{3\,Z\,Z^{-1}}{Z-(-2)}$$

$\downarrow$ Z.T

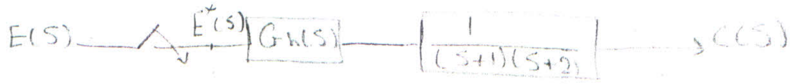
$$X(K) = 2(-1)^{K-1}\,u(K-1) - 3\,(-2)^{K-1}\,u(K-1)$$

$$\frac{1}{s} \rightarrow 1 \rightarrow \frac{z}{z-1} \qquad \frac{1}{s+1} \rightarrow e^{-T} \rightarrow \frac{z}{z-e^{-T}}$$

$$\frac{1}{s^2} \rightarrow t \rightarrow \frac{Tz}{(z-1)^2} \qquad \frac{1}{s+2} \rightarrow e^{-2T} \rightarrow \frac{z}{z-e^{-2T}}$$

* Sheet 2 (1)

1 Find the T.F $\frac{C(z)}{E(z)}$



$$\frac{C(z)}{E(z)} = Z\left[\frac{1-e^{-Ts}}{s(s+1)(s+2)}\right] = 1-z^{-1}\, Z\left[\frac{1}{s(s+1)(s+2)}\right] = [1-z^{-1}]\, Z\left[\frac{0.5}{s} - \frac{1}{s+1} + \frac{0.5}{s+2}\right]$$

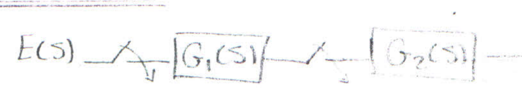
$$= \frac{z-1}{z}\left[\frac{0.5\,z}{z-1} - \frac{z}{z-e^{-T}} + \frac{0.5\,z}{z-e^{-2T}}\right] = 0.5 - \frac{z-1}{(z-e^{-T})} + \frac{0.5\,(z-1)}{(z-e^{-2})}$$

$\boxed{2}$ if $D(z) = \frac{z}{z-1}$ find TF $\longrightarrow D(S) = \frac{1}{S}$

$$\frac{C(z)}{E(z)} = Z\left[\frac{1-e^{-Ts}}{s^2(s+1)}\right] = \frac{z-1}{z}\, Z\left[\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1}\right]$$

$$= \frac{z-1}{z}\left[\frac{A\,z}{z-1} + \frac{B\,z}{(z-1)^2} + \frac{C\,z}{z-e^{-T}}\right]$$

$\boxed{3}$ If $G_1(S) = G_2(S) = \frac{1-e^{-TS}}{S(S+1)}$ find $\frac{C(z)}{E(z)}$



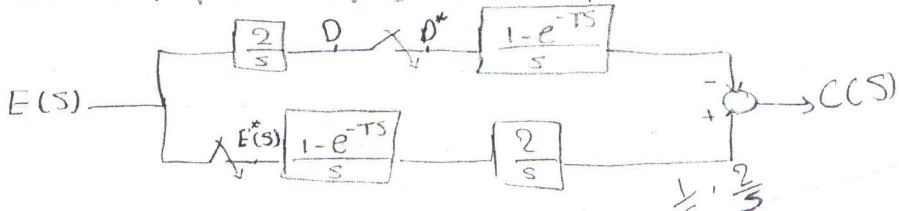
$$\frac{C(z)}{E(z)} = G_1(z)\cdot G_2(z) = Z\left[\frac{1-e^{-Ts}}{s(s+1)}\right]\cdot Z\left[\frac{1-e^{-Ts}}{s(s+1)}\right]$$

$$= \frac{z-1}{z}\, Z\left[\frac{1}{s} - \frac{1}{s+1}\right]\cdot \frac{z-1}{z}\, Z\left[\frac{1}{s} - \frac{1}{s+1}\right]$$

$$= \frac{z-1}{z}\left[\frac{z}{z-1} - \frac{z}{z-e^{-1}}\right]\cdot \frac{z-1}{z}\left[\frac{z}{z-1} - \frac{z}{z-e^{-1}}\right]$$

$$= \left[1 - \frac{z-1}{z-e^{-1}}\right]^2$$

$\boxed{4}$ Find $c(k)$ for $e(t)$ is unit step



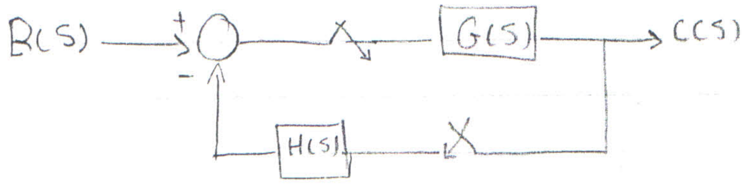
$\frac{1}{s}, \frac{2}{s}$

سؤال

$$C(z) = Z\left[\frac{(1-e^{-Ts})\,2}{s^2}\right] Z[E(s)] - Z\left[E(s)\,\frac{2}{s}\right]\cdot Z\left[\frac{1-e^{-Ts}}{s}\right]$$

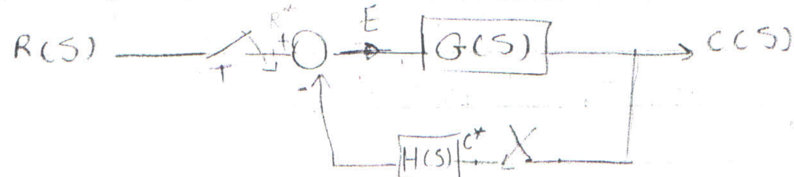
$$= 2\,\frac{z-1}{z}\,\frac{z}{(z-1)^2}\,\frac{z}{z-1} - 2\,\frac{z\,(z)}{(z-1)^2}\cdot\frac{z-1}{z}\cdot\frac{z}{z-1} = \frac{2z-2z^2}{(z-1)^2}$$

5 (a) Find the C(Z) for the System



$$C(Z) = \frac{G(Z)}{1 + G(Z)\cdot H(Z)} \cdot R(Z)$$

(b)



$$C(S) = G(S)\left[R^*(S) - H(S)\cdot C^*(S) \right]$$

$$(1 + H)\, C^*(S) = G^*(S)\, R^*(S)$$

$$C(S) = G(S)\left(R^*(S) - H(S)\, C^*(S)\right).$$

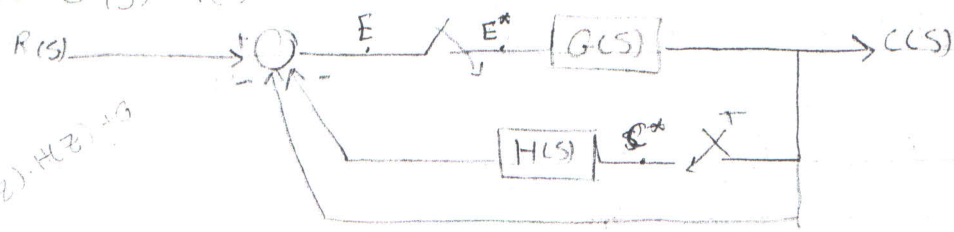
$$C^*(S) = \frac{G^*(S)}{1 + H^*(S)}\; R^*(S)$$

$$C^*(S) = G^*(S)\left(R^*(S) - H^*\, C^*(S)\right)$$

$$C(S) = E(S)\, G(S)$$

$$C^*(S) = E^*(S)\, G^*(S)$$

(C)



$$\frac{G(Z)}{1 + G(Z)\cdot H(Z) + G}$$

$$C(S) = E^*(S)\cdot G(S) \longrightarrow$$

$$E(S) = R(S) - H(S)\cdot C^*(S) - C(S)$$

$$E(S) = R(S) - H(S)\cdot C^*(S) - E^*(S)\cdot G(S) \longrightarrow$$

Staring

$$C^*(S) = E^*(S)\cdot G^*(S) \qquad \longrightarrow E^*(S) = \frac{C^*(S)}{G^*(S)}$$

$$E^*(S) = R^*(S) - H^*(S)\cdot C^*(S) - E^*(S)\cdot G^*(S)$$

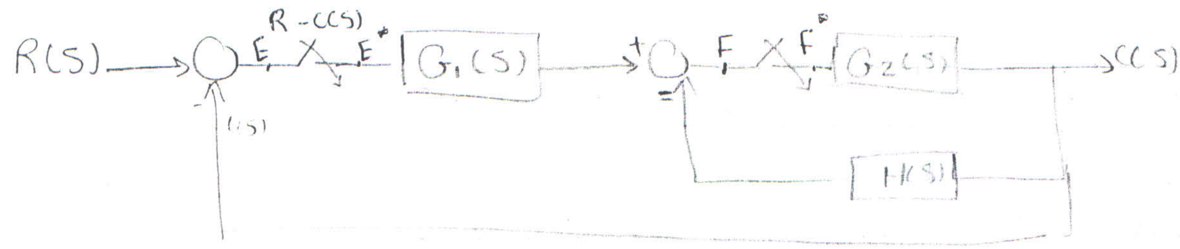
$$: E^*(S)\left[1 + G^*(S)\right] = R^*(S) - H^*(S)\cdot C^*(S).$$

$$C^*(S)\left[1 + G^*(S)\right] = R^*(S)\cdot G^*(S) - H^*(S)\cdot C^*(S)\cdot G^*(S)$$

$$C^*(S) = \frac{G^*(S)\cdot R^*(S)}{1 + G^*(S) + G^*(S)\cdot H^*(S).}$$

#sheet 2 (2)

[6] Find the overall T.F $= \frac{C(Z)}{R(Z)}$



$C(S) = F^*(S) . G_2(S)$

$E(S) = R(S) - C(S)$

$F(S) = G_1(S) . E^*(S) - C(S) . H(S).$

$\therefore E(S) = R(S) - F^*(S) . G_2(S).$

$F(S) = G_1(S) . E^*(S) - F^*(S) . H(S) . G_2(S)$

↓ Staring

$C^*(S) = F^*(S) . G_2^*(S).$ $\longrightarrow F^*(S) = \frac{C^*(S)}{G_2^*(S)}$

$E^*(S) = R^*(S) - F^*(S) . G_2^*(S)$

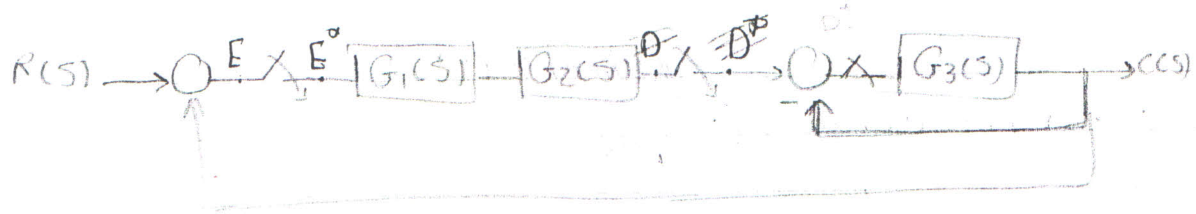
$F^*(S) = G_1^*(S) . E^*(S) - F^*(S) . \overline{HG_2(S)}^*$

$F^*(S) \left[1 + \overline{HG_2(S)}^* \right] = G_1^*(S) . E^*(S)$

$F^*(S) \left[1 + \overline{HG_2(S)}^* \right] = G_1^*(S) R^*(S) - G_1^*(S) G_2^*(S) . F^*(S)$

$F^*(S) \left[1 + HG_2(S)^* + G_1^*(S) . G_2^*(S) . F^*(S) \right] = G_1^*(S) . R^*(S)$

$C^*(S) \left[1 + HG_2(S)^* + G_1^* G_2^* F^*(S) \right] = G_1^* G_2^* R^*(S)$

(b) 

$$C(S) = D^*(S) \cdot G_3(S)$$

$$D(S) = G_1(S) \cdot G_2(S) \cdot E^*(S) - C(S)$$

$$E(S) = R(S) - C(S)$$

$$D(S) = G_1G_2(S)\, E^*(S) - D^*(S) \cdot G_3(S)$$

$$E(S) = R(S) - D^*(S) \cdot G_3(S)$$

starring

$$C^*(S) = D^*(S) \cdot G_3^*(S) \longrightarrow D^*(S) = C^*(S) / G_3^*(S)$$

$$D^*(S) = \overline{G_1G_2(S)}^*\, E^*(S) - D^*(S) \cdot G_3^*(S)$$

$$E^*(S) = R^*(S) - D^*(S) \cdot G_3^*(S)$$

$$D^*(S)\left[1 + G_3^*(S)\right] = \overline{G_1G_2(S)}^*\, E^*(S)$$

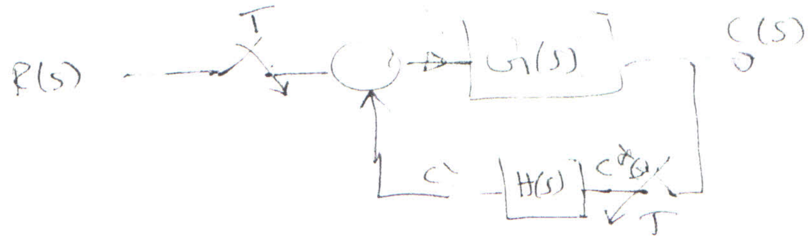
$$D^*(S)\left[1 + G_3^*(S)\right] = \overline{G_1G_2(S)}^*\, R^*(S) - \overline{G_1G_2(S)}^*\, G_3^*(S) \cdot D^*(S)$$

$$D^*(S)\left[1 + G_3^*(S) + \overline{G_1G_2(S)}^* \cdot G_3^*(S)\right] \cancel{D^*(S)} = \overline{G_1G_2}^*(S) \cdot R^*(S)$$

$$C^*(S)\left[1 + G_3^*(S) + \overline{G_1G_2(S)}^*\, G_3^*(S)\right] = \overline{G_1G_2(S)}^* \cdot G_3^*(S)\, R^*(S)$$

sheet(2) (3)

5. b



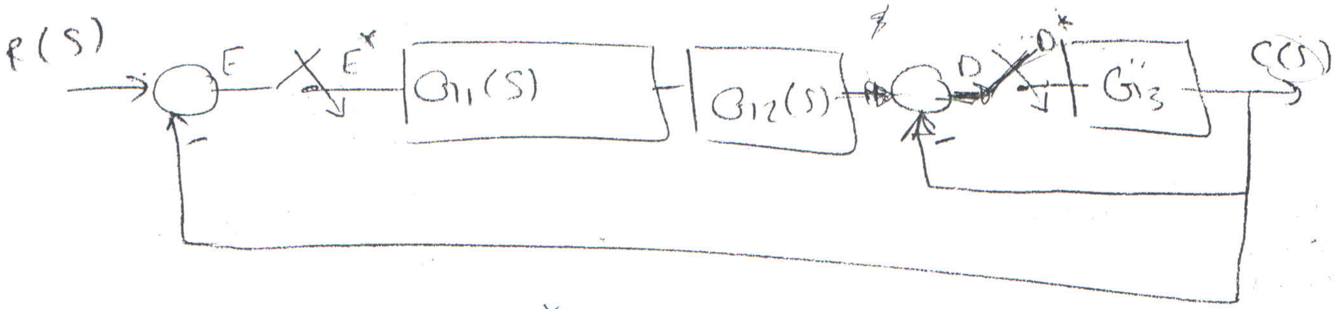
$$C(s) = G(s)\left[R^*(s) - C^*(s)H(s)\right]$$

$$C(s) = G(s)R^*(s) - C^*(s)H(s)G(s)$$

$$C^*(s) = G^*(s)R^*(s) - C^*(s)\overline{GH}^*(s)$$

$$\left(1 + \overline{GH}^*(s)\right)C^*(s) = G^*(s)R^*(s)$$

$$\frac{C^*(s)}{R^*(s)} = \frac{G^*(s)}{1 + \overline{GH}^*(s)}$$



$C(S) = G_3 D^*$

$D = G_1(S)\, G_2(S)\, E^* - C(S)$

$E = R(S) - C(S)$.

$C^*(S) = G_3^*\, D^* \longrightarrow$

$$\frac{C(Z)}{R(Z)} = \frac{G_1G_2(Z)\, G_3(Z)}{1 + G_1G_2(Z)\,G_3(Z) + G_3(}$$

$D^* = \overline{G_1G_2}^*\, E^* - C^*(S)$

$E^* = R^* - C^*(S)$

$\cancel{E^* = R^* - G_3^* D^*}$

$\cancel{E^* = R^* - \overline{G_1G_2}^*\, G_3^* \left[E^* - G_3^* C^*(S)\right]}$

$\cancel{\left[1 + \overline{G_1G_2G_3}^*\right] E^* = R^* - G_3^*\, C^*(S)}$

$E^* = R^* - G_3^*$

$D^* + C^*(S) = \overline{G_1G_2}^*\left(R^* - C^*\right)$

$\frac{C^*(S)}{G^*(S)} + C^*(S) = \overline{G_1G_2}^*\left(R^* - C^*\right)$

$C^*(S) + G^*(S)\, C^* =$

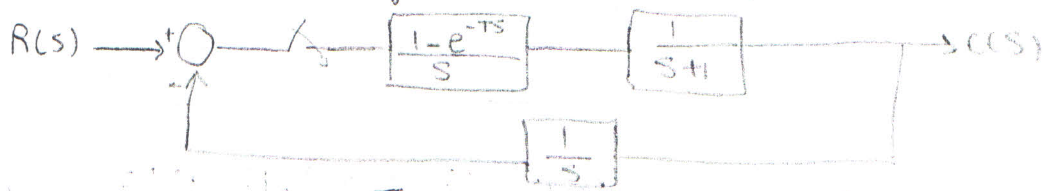
$\overline{G_1G_2G_3}\, R^* -$

$\overline{G_1G_2G_3}\, C^*$

$C^*\left(1 + G_3^*(S) + \overline{G_1G_2G_3}(S)\right) = \overline{G_1G_2G_3}$

Sheet(3) (1)

I Determine the o/p c(k) for the unit step i/p



$$T.F = \frac{C(Z)}{R(Z)} = \frac{Z\left[\frac{1-e^{-TS}}{s(S+1)}\right]}{1+Z\left[\frac{1-e^{-TS}}{S^2(S+1)}\right]} = \frac{(1-Z^{-1})Z\left[\frac{A}{S}+\frac{B}{S+1}\right]}{1+(1-Z^{-1})Z\left[\frac{A}{S}+\frac{B}{S^2}+\frac{C}{S+1}\right]}$$

$-A+1+1=1$

$$= \frac{(1-Z^{-1})Z\left[\frac{1}{S}-\frac{1}{S+1}\right]}{1+(1-Z^{-1})Z\left[\frac{1}{S}+\frac{1}{S+1}+\frac{1}{S^2}\right]} = \frac{\frac{Z-1}{Z}\left[\frac{Z}{Z-1}-\frac{Z}{Z-e^{-1}}\right]}{1+\frac{Z-1}{Z}\left[\frac{Z}{Z-1}+\frac{Z}{Z-e^{-1}}+\frac{Z}{(Z-1)^2}\right]}$$

$$= \frac{1-\frac{Z-1}{Z-e^{-1}}}{1+\left[1+\frac{Z-1}{Z-e^{-1}}+\frac{1}{Z-1}\right]} = \frac{\frac{\cancel{Z}-e^{-1}-\cancel{Z}+1}{Z-e^{-1}}}{1+\left[\frac{(Z-e^{-1})(Z-1)+(Z-1)^2+(Z-e^{-1})}{(Z-e^{-1})(Z-1)}\right]}$$

$$= \frac{\frac{1-e^{-1}}{Z-e^{-1}}}{\frac{(Z-e^{-1})(Z-1)+(Z-e^{-1})(Z-1)+(Z-1)^2+(Z-e^{-1})}{(Z-e^{-1})(Z-1)}} = \frac{(1-e^{-1})(Z-1)}{(Z-e^{-1})(Z-1)+(Z-e^{-1})(Z-1)+(Z-1)^2+(Z-e^{-1})}$$

$$C(Z) = \frac{0.632(Z-1)}{Z^2-Z+0.632}R(Z)$$

$$C(Z) = \frac{0.632(Z-1)}{Z^2-Z+0.632}\cdot\frac{Z}{Z-1} = \frac{0.632\,Z\,[a\sin w]}{Z^2-Z+0.632\,[a\sin w]}$$

$$\sin(wt) \xrightarrow{Z.T} \frac{Z\sin wT}{Z^2-2Z\cos wT+1}$$

$$a^t\sin(wt) \xrightarrow{Z.T} \frac{Z/a\,\sin wt}{(Z/a)^2-2(Z/a)\cos wT+1} = \frac{a\sin w\cdot Z}{Z^2-2a\cos w\,Z+a^2}$$

$a^2 = 0.632 \longrightarrow a = 0.796$

$2a\cos w = 1 \longrightarrow w = 51.027 = 0.89\,rad$

$a\sin w = 0.618$

$C(Z) =$

$\overline{GH(Z)}$ $\qquad$ Ch. eqn $= 1 + 0.L.T.F$

$= \overline{GH(Z)}$

[2] Determine the steady-state error when A

1- unit step I/P

$$O.L.T.F = \overline{GH(Z)}_Z = Z\left[\frac{1-e^{-TS}}{S^2(S+1)}\right] = \frac{Z-1}{Z}\, Z\left[\frac{A}{S} + \frac{B}{S^2} + \frac{S}{S+1}\right]$$

$$\frac{Z-1}{Z}\, Z\left[\frac{1}{S^2} + \frac{1}{S+1} - \frac{1}{S}\right] = \frac{Z-1}{Z}\left[\frac{Z}{Z-0.367} + \frac{Z}{(Z-1)^2} - \frac{Z}{Z-1}\right]$$

$$= \frac{Z-1}{Z-0.367} + \frac{1}{Z-1} - 1 = \frac{(Z-1)^2 + (Z-0.367) - (Z-1)(Z-0.367)}{(Z-1)(Z-0.367)}$$

$$\overline{GH(Z)}_Z = \frac{Z^2 - 2Z + 1 + Z - 0.367 - Z^2 + 1.367Z + 0.367}{(Z-1)(Z-0.367)} = \frac{0.367Z + 1}{(Z-1)(Z-0.367)}$$

$$e_{ss}\Big|_{\text{unit step}} = \cancel{\lim_{Z\to 1}} \frac{1}{1+K_P} = 0 \qquad = 0$$

$$K_V = \lim_{Z\to 1} (Z-1)\, \overline{GH(Z)} = \frac{1.367}{1-0.367} = \checkmark \qquad e_{ss} = \frac{1}{K_V}$$

Steady state O/P for unit step I/P

$$T.F = \frac{C(Z)}{R(Z)} = \frac{GH(Z)}{1+GH(Z)} = \checkmark$$

$$\therefore C(Z) = \checkmark \cdot \frac{Z}{Z-1}$$

$$\therefore C_{ss} = \lim_{Z\to 1} (Z-1)\, C(Z)$$

$$\frac{(Z-e^{-1})(Z-1) + (Z-1)^2 + (Z-e^{-1})}{(Z-e^{-1})(Z-1)}$$

$$\frac{Z^2 - Z + 0.632 - Z^2 + 2Z + 1 + Z - 0.367}{Z^2 - Z + 0.632}$$

$(Z-0.367)(Z-1)$

$Z^2 - 1.367Z + 0.367$

$Z^2 - 1.367Z + 0.367 - Z^2 + 2Z - 1 + Z - 0.367$

$$\frac{1.633Z - 1}{Z^2 - 1.367Z + 0.367}$$

sheet (3) (2)

$\boxed{3}$ $\frac{C(Z)}{R(Z)} = \frac{0.4}{S(S+1)}$

ⓐ impulse response $\longrightarrow$, $R(Z) = 1$

$C(Z) = \frac{0.4}{S(S+1)} R(Z) = \cancel{\frac{A}{Z}} + \cancel{B} = 0.4\, Z\left[\frac{A}{S} + \frac{B}{S+1}\right] = 0.4\, Z\left[\frac{1}{S} - \frac{1}{S+}\right.$

$= 0.4\left[\frac{Z}{Z-1} - \frac{Z}{Z-0.367}\right] = \frac{0.4Z(Z-0.367) - 0.4Z(Z-1)}{(Z-1)(Z-0.367)}$

$C(K) = 0.4\; U(K) - 0.4\,(0.367)^{K-1}$

ⓑ Discus the stability $|Z| < 1$ $Z = 1, 0.367$

$= \lim_{Z \to 1} \cancel{(Z-1)} \frac{0.4\;(Z-0.367) - 0.4Z(Z-1)}{\cancel{(Z-1)}(Z-0.367)} = \frac{0.2532}{0.633} = 0.4$

critical stable.

$\boxed{4}$ ⓐ O.L.T.f $= \cancel{\frac{C(Z)}{R(Z)}} = \underline{\frac{Z-1}{Z} Z\left[\frac{K}{S^2(S+1)}\right]} = K \frac{0.367Z+1}{(Z-1)(Z-0.367)}$

※Sheet 4 (1)

1 Check if the roots of the following chr eqs. lie within the unit circle.

a) $5Z^2 - 2Z + 3 = 0$

$(Z - (0.2 + 0.75i))(Z - (0.2 - 0.75i)) = 0$

$|Z_{1,2}| = \sqrt{(0.2)^2 + (0.75)^2} = 0.776 < 1$

The system is stable → lie on unit circle.

b) $Z^3 - 0.2Z^2 - 0.25Z + 0.05 = 0$

$(Z + 0.5)(Z - 0.5)(Z - 0.2) = 0$

$|Z_1| = +0.5 \qquad |Z_2| = 0.5 \qquad |Z_3| = 0.2$

all poles located in unit Circle

the system is stable.

c) $Z^4 - 1.7Z^3 + 1.04Z^2 - 0.268Z + 0.024 = 0$

$\cancel{(Z^4 - 1.7Z^3)}$ - using Jury Test

(1) $F(1) \rightarrow 1 - 1.7 + 1.04 - 0.268 + 0.024 = 0.096 > 0$

(2) $(-1)^4 F(-1) \rightarrow 1 + 1.7 + 1.04 + 0.268 + 0.024 = 4.032 > 0$

(3) $|a_0| = 0.024 < |a_n| = 1$

(4) Jury Test

$b_0 = \begin{vmatrix} 0.024 & 1 \\ 1 & 0.024 \end{vmatrix} = -0.99$

$b_1 = \begin{vmatrix} 0.024 & -1.7 \\ 1 & -0.268 \end{vmatrix} = 1.693$

$b_2 = \begin{vmatrix} 0.024 & 1.04 \\ 1 & 1.04 \end{vmatrix} = -1.05$

$b_3 = \begin{vmatrix} 0.024 & -0.268 \\ 1 & -1.7 \end{vmatrix} = 0.227$

$c_0 = \begin{vmatrix} -0.99 & 0.227 \\ 0.227 & -0.99 \end{vmatrix} = 0.946$

$c_2 = \begin{vmatrix} -0.99 & 1.693 \\ 0.227 & -1.015 \end{vmatrix} = 0.629$

	Z^0	Z^1	Z^2	Z^3	Z^4
1	0.024	-0.268	1.04	-1.7	1
2	1	-1.7	1.04	-0.268	0.024
3	b_0	b_1	b_2	b_3	
4	b_3	b_2	b_1	b_0	
5	C_0	C_1	C_2		

(5) $|b_0| = 0.99 > |b_3| = 0.22$

$|C_0| = 0.946 > |C_2| = 0.629$

The system is stable

all roots lie on unit circle

(d) $Z^3 + 5Z^2 + 3Z + 2 = 0$

$(Z + 4.42)(Z - (-0.29 + 0.61i))(Z + (0.29 + 0.61i)) = 0$

$|Z_1| = 4.42 < 1$

$|Z_{3,2}| = \sqrt{(0.29)^2 + (0.61)^2} = 0.675 < 1$

the pole lie on unit circle

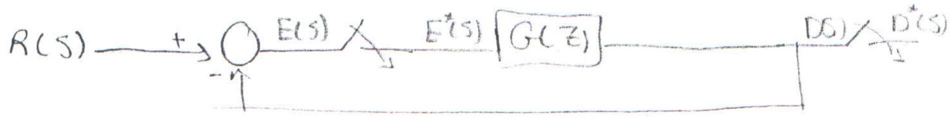
the system is stable.

sheet 4 (2)

[2] For the system shown with $G(Z) = \frac{0.1(Z+0.9)}{(Z-1)(Z-0.7)}$

Check the stability using Jury, bilinear

$T.F = \frac{G(Z)}{1+G(Z)}$



$Ch. eqn = 1 + GH(z) = 0$

$\therefore (Z-1)(Z-0.7) + 0.1Z + 0.09 = 0$

$Z^2 - 1.7Z + 0.7 + 0.1Z + 0.09 = 0$

$Z^2 - 1.6Z + 0.79 = 0$

[1] Bilinear $\quad Z = \frac{1+r}{1-r}$

$\left(\frac{1+r}{1-r}\right)^2 - 1.6\left(\frac{1+r}{1-r}\right) + 0.79 = 0 \qquad 1-2r+r^2$

$(1+r)^2 - (1.6)(1-r^2) + 0.79(1-r)^2 = 0$

$1+2r+r^2 - 1.6 + 1.6r^2 + 0.79 - 1.58r + 0.79r^2 = 0$

$3.39r^2 + 1.58r + 0.19 = 0$

r^2	3.39	0.19
r	1.58	0
r^0	0.19	

$\longrightarrow$ stable

[2] Jury Test

Ch. eqn $\rightarrow Z^2 - 1.6Z + 0.79 = 0$

1) $F(1) \quad 1 - 1.6 + 0.79 = 0.19 > 0$ ✓

2) $(-1)^2 f(-1) \quad 1 + 1.6 + 0.79 = 3.39 > 0$ ✓

3) $|a_0| = 0.79 < |a_1| = 1$ ✓

the system is stable.

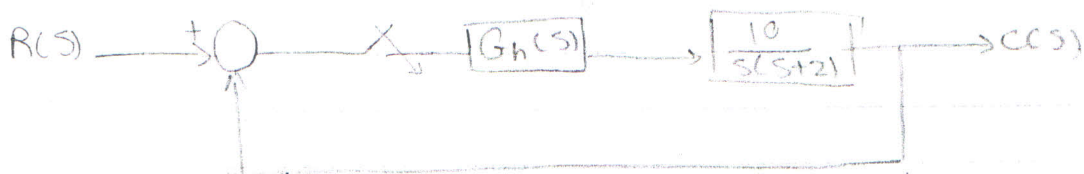
$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2}$

$B = 0.5 \quad C = 0.25$

at $s^2 = -1$

$-A + \frac{1}{2} + \frac{1}{4} = 1 \quad A = -0.25$

[3] write the ch/eqn for the System, Check Stability



$$O.L.T.F = 10 Z\left[\frac{1-e^{-TS}}{s^2(s+2)}\right] = 10(1-Z^{-1}) Z\left[\frac{A^{1/2}}{s} + \frac{B^{1/2}}{s^2} + \frac{C^{1/4}}{s+2}\right] \quad \frac{1}{s^2(s+2)}$$

$$= 10(1-Z^{-1}) Z[A\,u(t) + B\,t + C\,e^{-2t}]$$

$$= 10\left[\frac{Z-1}{Z}\right]\left[\frac{AZ}{Z-1} + \frac{BZ}{(Z-1)^2} + \frac{CZ}{Z-e^{-2}}\right]$$

$$= 10\left[A^{1/4} + \frac{B^{1/2}}{(Z-1)} + \frac{C^{1/4}(Z-1)}{(Z-e^{-2})}\right] = -5/2 + \frac{5}{Z-1} + \frac{5}{2}\frac{(Z-1)}{(Z-e^{-2})}$$

$$= \frac{-2.5(Z-1)(Z-0.135) + 5(Z-0.135) + 2.5(Z-1)^2}{(Z-1)(Z-0.135)} \quad Z^2 - 2Z + 1$$

$$= \frac{-2.5Z^2 + 2.83Z - 0.3375 + 5Z - 0.675 + 2.5Z^2 - 5Z + 2.5}{Z^2 + 1.135Z + 0.135}$$

$$= \frac{2.83Z + 2.2}{(Z-1)(Z-0.135)}$$

ch/c eqn $= 1 + O.L.T.F = (Z-1)(Z-0.135) + 2.83Z + 2.2 = 0$

$= Z^2 - 1.135Z + 0.135 + 2.83Z + 2.2$

$F(Z) = Z^2 + 1.7Z + 2.335$

Jury Test

1. $F(1) = 1 + 1.7 + 2.335 > 0$ ✓
2. $(-1)^2 F(-1) = 1 - 1.7 + 2.335 > 0$ ✓
3. $|a_0| = 2.335 < |a_n| = 1$ ✓

the system is stable.

Ch/c eqn = 1 + O.L.T.F

1 + GH(Z)   sheet 4 (3)

[4] for the following system, Determine the Rang of K for stability

(a)

$$\cancel{\frac{C(S)}{R(S)}} = O.L.T.F = GH(Z) = Z\left[\frac{(1-e^{-TS})K}{S^2(S+2)}\right] = (1-Z^{-1})Z\left[\frac{K}{S^2(S+2)}\right]$$

$$= K\,(1-Z^{-1})Z\left[\frac{\overset{-0.25}{A}}{S} + \frac{\overset{1/2}{B}}{S^2} + \frac{\overset{1/4}{C}}{S+2}\right] = K(1-Z^{-1})Z\left[\frac{-0.25}{S} + \frac{0.5}{S^2} + \frac{0.25}{S+2}\right]$$

$\frac{Z-1}{Z}$

$$= K(1-Z^{-1})\left[\frac{-0.25\,Z}{Z-1} + \frac{0.5\,Z}{(Z-1)^2} + \frac{0.25\,Z}{Z-e^{-2}}\right]$$

$$= K(\cancel{1-Z^{-1}})\left[-0.25 + \frac{0.5}{(Z-1)} + \frac{0.25(Z-1)}{Z-e^{-2}}\right]$$

Z^2-2Z+1

$$= K\,\frac{-0.25(Z-1)(Z-e^{-2}) + 0.5(Z-e^{-2}) + 0.25(Z-1)^2}{(Z-1)(Z-e^{-2})} \qquad e^{-2} = 0.135$$

$.25Z + 0.0337$

$$= K\,\frac{-0.25(Z^2 - Z - e^{-2}Z + e^{-2}) + 0.5Z - 0.5e^{-2} + 0.25Z^2 - 0.5Z + 0.25}{(Z-1)(Z-e^{-2})}$$

$$= K\,\frac{-0.25Z^2 + 0.25Z + 0.0337Z - \overset{0.0337}{} + 0.5Z - 0.067 + 0.25Z^2 - 0.5Z + 0.25}{(Z-1)(Z-e^{-2})}$$

$$= K\,\frac{0.2837Z + 0.2167}{(Z-1)(Z-0.135)}$$

Ch/c eqn $\longrightarrow 1 + O.L.T.F = 1 + \frac{K(0.2837Z + 0.2167)}{(Z-1)(Z-0.135)} = 0$

$(Z-1)(Z-0.135) + K(0.2837Z + 0.2167)$

$Z^2 - 1.135Z + 0.135 + 0.2837KZ + 0.2167K = 0$

$Z^2 + Z[0.2837K - 1.135] + 0.135 + 0.2167K = 0$

(1) $f(1) > 0 \longrightarrow 1 + 0.2837K - 1.135 + 0.135 + 0.2167K > 0$

$0.5004K > 0 \qquad K > 0$

(2) $(-1)^2 f(-1) > 0 \longrightarrow 1 - 0.2837K + 1.135 + 0.135 + 0.2167K > 0$

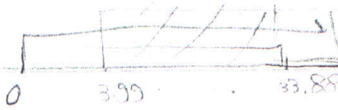
$2.27 - 0.067K > 0 \longrightarrow 0.067K < 2.27$

$(K < 33.881)$

(3) $|a_0| = 1 < |a_n| = 0.135 + 0.2167K$

$0.135 + 0.2167K > 1 \qquad K > 3.992$

$3.88 < K < 3.99$



(b) O.L.T.F $= Z\left[\frac{(1-e^{-TS})K(e^{-TS})}{S^2(S+1)}\right] = K(1-Z^{-1})\, Z\left[\frac{e^{-TS} - e^{-2TS}}{S^2(S+1)}\right]$

$1 - e^{-TS} = 1 - z^{-1}$

$K(1-z^{-1})\, z^{-1} \quad Z\left[\frac{1}{S^2(S+1)}\right]$

*Sheet(5) (1)

[II] Calculate the e_{ss} for a unit step, unit ramp i/p for O.L.T.F.

(a) $GH(z) = \dfrac{0.0952\,K\,z}{(z-1)(z-0.965)}$ Type 1

$$e_{ss} = \lim_{z \to 1} (z-1) \frac{R(z)}{1+GH(z)}$$

1- for unit step i/p $\longrightarrow \cancel{\frac{1}{1+}} \frac{1}{K_p}$

$$K_p = \lim_{z \to 1} GH(z) = \lim_{z \to 1} \frac{0.0952\,Kz}{(z-1)(z-0.965)} = \infty$$

$$\therefore e_{ss} = 1 + \frac{1}{\infty} = 0 \qquad e_{ss} = \frac{1}{\infty} = 0$$

2- for unit ramp i/p $\longrightarrow \frac{1}{K_V}$

$$K_V = \frac{1}{T} \lim_{z \to 1} (z-1)\,GH(z) = \lim_{z \to 1} \frac{0.0952\,Kz}{z-0.965} = \frac{0.0952\,K}{0.035} = 2.72K$$

$$\therefore e_{ss} = \frac{1}{2.72K}$$

3- for $r(t) = 3u(t) + 2t \longrightarrow R(z) = \dfrac{3z}{z-1} + 2\dfrac{Tz}{(z-1)^2}$

$$e_{ss}\Big|_{r(t)=3u(t)+2t} = 3\left(\frac{1}{1+K_p}\right) + 2\left(\frac{1}{K_V}\right)$$

$$= 3 * 0 + 2\,\frac{1}{2.72K} = \frac{25}{34K}$$

$\frac{1}{1+K_p}$	∞	∞
0	$\frac{1}{K_v}$	∞
0	0	$\frac{1}{K_a}$

(b) $GH(Z) = \dfrac{Z+0.9}{(Z)(Z-0.5)}$ $\longrightarrow$ Type (0)

1- for unit step i/p $\longrightarrow e_{ss} = \dfrac{1}{1+K_p}$

$$K_p = \lim_{Z\to 1} \frac{Z+0.9}{Z(Z-0.5)} = \frac{1.9}{0.5} = 3.8$$

$$e_{ss} = \frac{1}{1+3.8} = \frac{5}{24} = 0.208$$

2- for unit ramp i/p $\longrightarrow e_{ss} = \dfrac{1}{K_v}$

$$K_v = \frac{1}{T} \lim_{Z\to 1} (Z-1) \frac{(Z+0.9)}{Z(Z-0.5)} = 0$$

$$e_{ss} = \frac{1}{0} = \infty$$

3- for $r(t) = 3u(t) + 2t$

$$e_{ss}\Big|_{r(t)=3u(t)+2t} = 3\left(\frac{1}{1+K_p}\right) + 2\left(\frac{1}{K_v}\right) = 3*0.208 + 2*\infty = \infty$$

(c) $GH(Z) = \dfrac{(Z+1)}{(1-Z)^2} = \dfrac{Z+1}{(-(Z-1))^2} = \dfrac{Z+1}{(Z-1)^2}$ $\longrightarrow$ Type (2)

1- for unit step $\longrightarrow e_{ss} = \dfrac{1}{1+K_p}$

$$K_p = \lim_{Z\to 1} \frac{Z+1}{(Z-1)^2} = \infty$$

$$\therefore e_{ss} = \frac{1}{1+\infty} = 0$$

2- for unit ramp i/p $\longrightarrow e_{ss} = \dfrac{1}{K_v}$

$$K_v = \frac{1}{T} \lim_{Z\to 1} \cancel{(Z-1)} \frac{(Z+1)}{(Z-1)^{\cancel{2}}} = \frac{2}{0} = \infty$$

$$e_{ss} = \frac{1}{\infty} = 0$$

3- for $e_{ss}\Big|_{r(t)=3u(t)+2t}$

$$= 3\left(\frac{1}{1+K_p}\right) + 2\left(\frac{1}{K_v}\right) = 0$$

sheet(5)(2)

(d) $GH(Z) = \dfrac{Z}{(Z^2-1)(Z^2-Z+0.5)}$

$= \dfrac{Z}{(Z-1)(Z+1)(0.5Z+0.5i)(0.5Z-0.5i)} \longrightarrow$ Type 1

1- for unit step i/p $\rightarrow ess = \dfrac{1}{1+Kp}$

$Kp = \lim\limits_{Z\to 1} GH(z) = \dfrac{1}{0} = \infty$

$\therefore ess = \dfrac{1}{1+\infty} = 0$

2- for unit ramp i/p $\rightarrow ess = \dfrac{1}{Kv}$

$Kv = \lim\limits_{Z\to 1} (Z-1)\, GH(z) = \dfrac{1}{(2)(0.5)} = 1$

$ess = \dfrac{1}{1} = 1$

3- for $r(t) = 3u(t) + 2t$

$ess|_{r(t)=3u(t)+2t} = 3\dfrac{1}{Kp+1} + 2\dfrac{1}{Kv} = 2$

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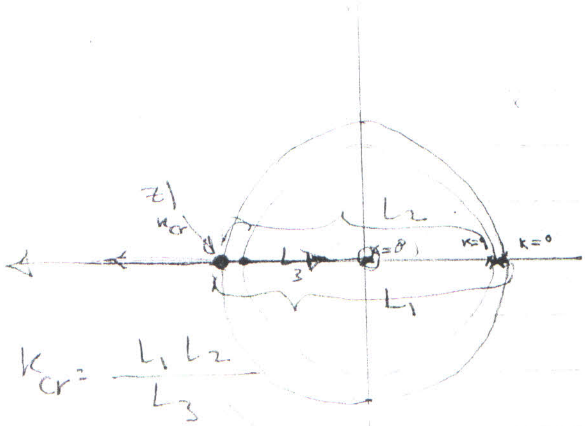
[2] Draw root-locus & calculate range of K for stability for the following o/p

[1] $GH(Z) = \dfrac{0.0952\, KZ}{(Z-1)(Z-0.905)}$



1) $np = 2 \rightarrow 1,\ 0.905$

$nz = 1 \rightarrow 0$

2) real part
1) $0.905 \longrightarrow 1$
2) $0 \longrightarrow -\infty$

3) Asymptotic lines

$No = np - nz = 1$

$\delta_o = \dfrac{\sum poles - \sum zeros}{1} = 1 + 0.905 - 0 = 1.905$

$K_{cr} = \dfrac{L_1 L_2}{L_3}$

$K = f(z)$

$\varphi = 180$

$\dfrac{dK}{dz} = 0$

4) Break Points

Ch/ eqn $\rightarrow 1 + GH(Z) = 0$

$$1 + \frac{0.0952KZ}{(Z-1)(Z-0.905)} = 0 \rightarrow (Z-1)(Z-0.905) + 0.0952KZ = 0$$

$$Z^2 - 1.905Z + 0.905 + 0.0952K\,Z = 0$$

$$K = -\frac{(Z^2 - 1.905Z + 0.905)}{0.0952Z} = -\left[10.5Z - 20.01 + 9.51Z^{-1}\right]$$

$$\frac{dK}{dZ} = 0 \longrightarrow 10.5 - \frac{9.51}{Z^2} = 0 \longrightarrow \frac{10.5Z^2 - 9.51}{Z^2} = 0$$

$$\therefore Z^2 = \frac{9.51}{10.5} \longrightarrow Z = +0.952 \;, -0.952$$

$$\therefore r = \frac{0.952 + 0.952}{2} = 0.952 \quad , \text{Center} = 0$$

system stable.

$\rightarrow$ Determining Kcr using jury

Ch/ eqn $\longrightarrow \underbrace{Z^2}_{a_2 = 1} + \underbrace{(0.952K - 1.905)}_{a_1} Z + \underbrace{0.905}_{a_0 =} = 0$

[1] $F(1) > 0$

$1 + 0.952K - 1.905 + 0.905 > 0$

$\boxed{K > 0}$

[2] $(-1)^2\, F(-1) > 0$

$-0.952K + 1.905 + 0.905 > 0 \longrightarrow -0.952K + 2.81 > 0$

$K < \frac{+2.81}{0.952}$ $\quad \boxed{K < 2.951}$

[3] $|a_0| < |a_n|$ ..

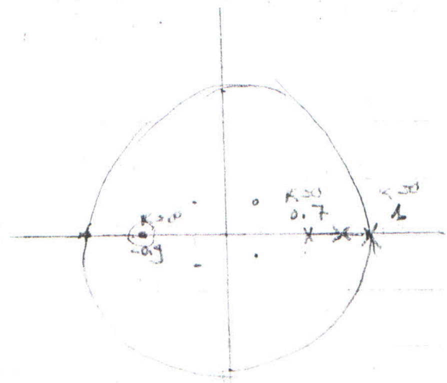
$0.905 < 1$

$0 < K < 2.951$

sheet(5) (3)

[2] $GH(z) = \frac{k(Z+0.9)}{(Z-1)(Z-0.7)}$

0.9 + 0.14 i



① $n_p = 2 \longrightarrow 1, 0.7$

$n_z = 1 \longrightarrow -0.9$

2) real part ① $0.7 \rightarrow 1$

② $-0.9 \rightarrow -\infty$

③ Asymptotic Lines

$N_o = 1$

$f_o = \frac{1.7+0.9}{1} = 2.6$

$Q = 180$

④ Break points

ch/eqn $\longrightarrow 1 + GH(Z) = 0$ $\quad 1 + K\,GH(z) = 0$

$(Z-1)(Z-0.7) + K(Z+0.9) = 0$ $\quad K\,GH(z) = -1 \quad K = \frac{-1}{GH(z)}$

$Z^2 - 1.7Z + 0.7 + K(Z+0.9) = 0$

$K = \frac{Z^2-1.7Z+0.7}{Z-0.9} \Rightarrow \frac{dK}{dz} = \frac{(Z-0.9)(2Z-1.7) - (Z^2-1.7Z+0.7)}{(Z-0.9)^2} = 0$

$\therefore 2Z^2 - 3.5Z + 1.53 - Z^2 + 1.7Z - 0.7 = 0$

$Z^2 - 1.8Z + 0.83 = 0 \qquad Z_{1,2} = 0.9 \pm 0.14i$

$K = \frac{-1(Z-1)(Z-0.7)}{(Z+0.9)}$ $\qquad \frac{dk}{dz} = 0$

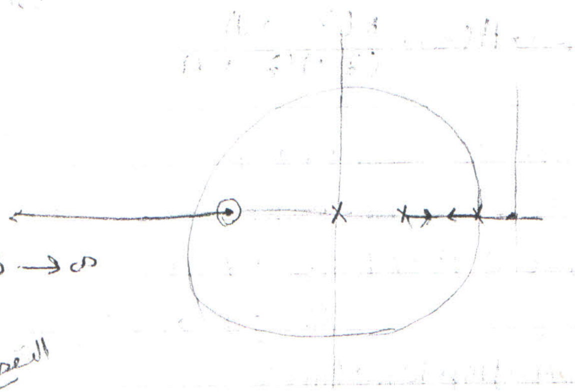
$z^2 - 1.7z + 0.7$

$\frac{dk}{dz} = \frac{(Z+0.9)(2Z-1.7) - (Z^2-1.7Z+0.7)(1)}{(Z+0.9)^2} = 0$

$(Z+0.9)(2Z-1.7) = Z^2 - 1.7Z + 0.7$

$Z =$

(5) $GH(Z) = \dfrac{0.15K(Z+0.7453)}{Z(Z-1)(Z-0.4119)}$



1) $n_p = 3 \qquad 0,\ 1,\ 0.4119$

$n_z = 1 \qquad -0.7453$

2) Real Part ① $1 \longrightarrow 0.4119$ ② $-0.7453 \rightarrow \infty$

3) asymptotic lines

① $N_o = 3-1 = 2$

$\sigma_o = \dfrac{0+1+0.4119+0.7453}{2} = 1.0786$ [illegible]

$\phi_o = -90,\ 90$

4) Break point $1+GH(Z)=0$

$Z(Z-1)(Z-0.4119) + 0.15KZ + 0.111K = 0$

$Z(Z^2 - 1.4119Z + 0.4119) + 0.15KZ + 0.111K = 0$

$Z^3 - 1.4119Z^2 + 0.4119Z + 0.15KZ + 0.111K = 0$

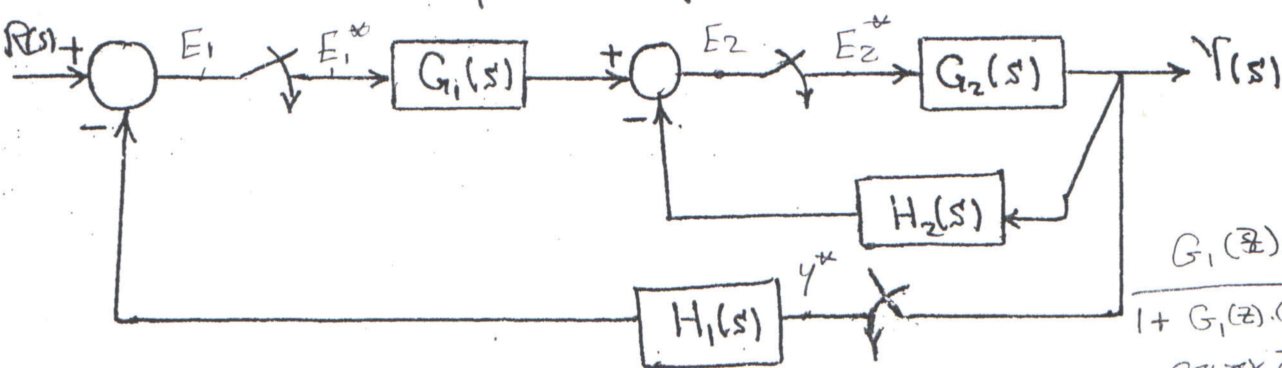
$Z^3 - 1.4119Z^2 + \cancel{0.5619Z}\ Z(0.4119 + 0.15K)Z + 0.111K = 0$

$K = -\dfrac{Z(Z-1)(Z-0.4119)}{(Z+0.7453)} = \dfrac{Z^3 - 1.4119Z^2 + 0.4119Z}{Z+0.7453}$

$\dfrac{dK}{dZ} = \dfrac{(Z+0.7453)(3Z^2 - 2.8238Z + 0.4119) - (Z^3 - 1.4119Z^2 + 0.4119Z)}{(Z+0.7453)^2}$

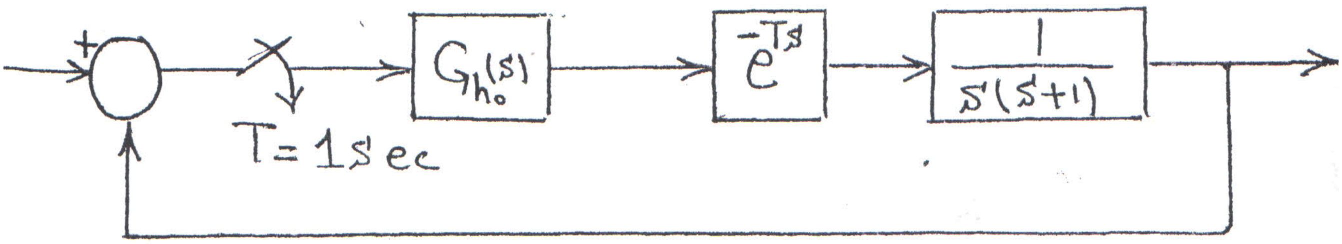
midteerm

1. For the following block diagram, Find, if it exists, the closed Loop transfer function.



$$\frac{G_1(z).G_2}{1+G_1(z).G_2(z)+\cancel{G_1(z)}.G_2H_2}$$

2. For the following unity feedback control systems,



(1) Determine the openloop transfer function.
(2) what is the type of the system and Calculate the error Constants.
10 (3) find the steady state error for unit step i/p.
(4) Determine the closed loop T.F.
(5) CaLcuLate the steady state value of the output for unit step i/p.
(6) check the system stability.

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sec : 1

※ Sheet (6) (1)

[I] Draw the Bode Diagram of the Following functions, Determine GM, PM.

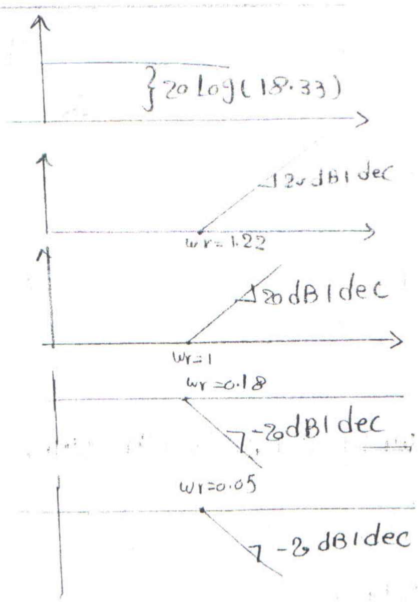
(a) $GH(Z) = \dfrac{0.5(Z+0.1)}{(Z-0.7)(Z-0.9)}$ $\dashrightarrow$ put $Z = \dfrac{1+r}{1-r}$

$$GH(r) = \frac{0.5\left(\frac{1+r}{1-r}+0.1\right)}{\left(\frac{1+r}{1-r}-0.7\right)\left(\frac{1+r}{1-r}-0.9\right)} = 0.5\,\frac{\frac{1+r+0.1-0.1r}{1-r}}{\frac{1+r-0.7+0.7r}{1-r}\cdot\frac{1+r-0.9+0.9r}{1-r}}$$

$$= 0.5\left[\frac{(1.1+0.9r)(1-r)}{(0.3+1.7r)(0.1+1.9r)}\right] = \frac{0.5\times0.1}{0.3\times0.1}\left[\frac{1+\frac{r}{1.22}}{\left(1+\frac{r}{0.18}\right)\left(1+\frac{r}{0.5}\right)}\right]$$

$$GH(jw) = 18.33\left[\frac{\left(1+\frac{jw}{1.22}\right)(1-jw)}{\left(1+\frac{jw}{0.18}\right)\left(1+\frac{jw}{0.5}\right)}\right]$$

Term	$Q(wr)$
18.33	0
$1+j\frac{wr}{1.22}$	$\tan^{-1}\frac{wr}{1.22}$
$1-jw$	$-\tan^{-1}(wr)$
$\frac{1}{1+j\frac{wr}{1.18}}$	$-\tan^{-1}\left(\frac{wr}{0.18}\right)$
$\frac{1}{1+j\frac{wr}{0.05}}$	$-\tan^{-1}\left(\frac{wr}{0.05}\right)$



$$Q(wr) = \tan^{-1}\left(\frac{wr}{1.22}\right) - \tan^{-1}(wr) - \tan^{-1}\left(\frac{wr}{0.18}\right) - \tan^{-1}\left(\frac{wr}{0.05}\right)$$

wr	0	1.22	1	0.18	0.05	∞
Q	0	-174.919	-172.593	-121.286	-61.039	-180

(b) $GH(z) = \dfrac{0.5(z-1)}{(z-0.1)(z-0.8)} \longrightarrow$ put $z = \dfrac{1+r}{1-r}$

$$GH(r) = \frac{0.5\left(\frac{1+r}{1-r} - 1\right)}{\left(\frac{1+r}{1-r} - 0.1\right)\left(\frac{1+r}{1-r} - 0.8\right)} = \frac{0.5(1+r-1+r)/1-r}{\frac{(1+r-0.1+0.1r)}{(1-r)}\frac{(1+r-0.8+0.8r)}{(1-r)}}$$

$$= \frac{0.5(2r)(1-r)}{(0.9+1.1r)(0.2+1.8r)} = 5.55\,\frac{r(1-r)}{\left(1+\frac{r}{0.818}\right)\left(1+\frac{r}{0.111}\right)}$$

$$GH(jw) = 5.55\,\frac{jw(1-jw)}{\left(1+\frac{jw}{0.818}\right)\left(1+\frac{jw}{0.111}\right)}$$

Term	$Q(w_r)$	
5.55	0	$\}20\log 5.55$
jw_r	$+90$	$+20\,dB/dec$, $w_r=1$
$1+jw_r$	$\tan^{-1} w_r$	$+20\,dB/dec$, $w_r=1$
$1/1+j\frac{w_r}{0.818}$	$-\tan^{-1}\frac{w_r}{0.818}$	$w_r=0.818$, $-20\,dB/dec$
$1/1+j\frac{w_r}{0.111}$	$-\tan^{-1}\frac{w_r}{0.111}$	$w_r=0.111$, $-20\,dB/dec$

$$Q(w_r) = 0 + 90 + \tan^{-1}(w_r) - \tan^{-1}\frac{w_r}{0.818} - \tan^{-1}\frac{w_r}{0.111}$$

31.8199 115.211 -65.53

w_r	0	1	0.818	0.111	0.5	1.5	2	∞
$Q(w_r)$	90	0.617	80.08	43.306	7.646	-0.8527	-1.143	

sheet (b) (2)

(c) $GH(Z) = \dfrac{0.5(Z+0.76)}{(Z-1)(Z-0.45)}$ $\longrightarrow$ Put $Z = \dfrac{1+r}{1-r}$

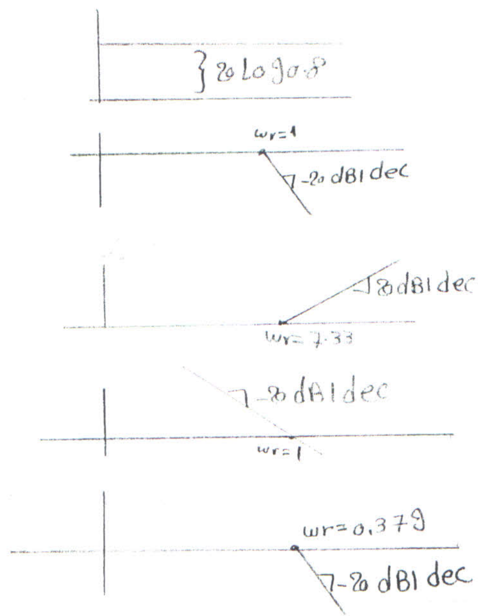
$$GH(r) = 0.5\,\frac{\left(\frac{1+r}{1-r}+0.76\right)}{\left(\frac{1+r}{1-r}-1\right)\left(\frac{1+r}{1-r}-0.45\right)} = 0.5\,\frac{(1+r+0.76-0.76r)(1-r)}{(1+r-1+r)(1+r-0.45+0.45r)}$$

$$= 0.5\,\frac{(1-r)(1.76+0.24r)}{2r\,(0.55+1.45r)} = \frac{0.5 \times 1.76}{2 \times 0.55}\;\frac{(1-r)\left(1+\frac{0.24}{1.76}r\right)}{\left(1+\frac{1.45}{0.55}r\right)r}$$

$$= 0.8\,\frac{(1-r)\left(1+\frac{r}{7.33}\right)}{r\left(1+\frac{r}{0.379}\right)}$$

$$GH(jw) = \frac{0.8\,(1-jw_r)\left(1+\frac{jw_r}{7.33}\right)}{jw_r\left(1+\frac{jw_r}{0.379}\right)}$$

Term	$\varphi(w_r)$
0.8	0
$1-jw_r$	$-\tan^{-1}\frac{w_r}{1}$
$1+j\frac{w_r}{7.33}$	$\tan^{-1}\frac{w_r}{7.33}$
$\frac{1}{jw_r}$	-90
$1/\left(1+\frac{jw_r}{0.379}\right)$	$-\tan^{-1}\frac{w_r}{0.379}$



$$\varphi(w_r) = -\tan^{-1} w_r - \tan^{-1}\frac{w_r}{0.379} + \tan^{-1}\frac{w_r}{7.33} - 90$$

w_r	0	1	0.379	7.33	0.5	5	∞
φ	-90	-196.625	-152.796	-214.271	-165.5	-220.05	-180

$z^3 F(z) - z^3 f(0) - z^2 f(2) - z f(2)$

sheet (6)(3)

[2] for the following system → Controlable, observable, Draw State Diagram

a) $y(k+2) + 6y(k+1) + 5y(k) = 2r(k)$

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$$z^2 Y(z) + 6zY(z) + 5Y(z) = 2R(z)$$

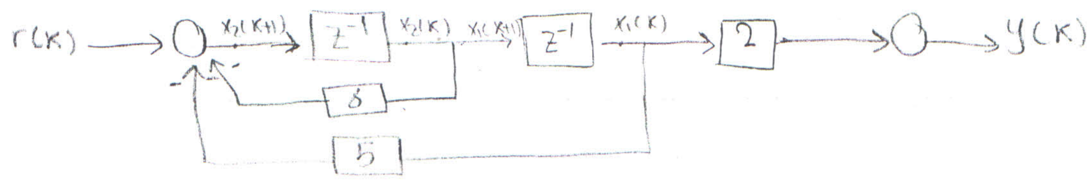
$$Y(z)[z^2 + 6z + 5] = 2R(z)$$

$$\frac{Y(z)}{R(z)} = \frac{2}{z^2+6z+5}$$

1) controlable form

$$X(k+1) = \begin{pmatrix} 0 & 1 \\ -5 & -6 \end{pmatrix} X(k) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r(k)$$

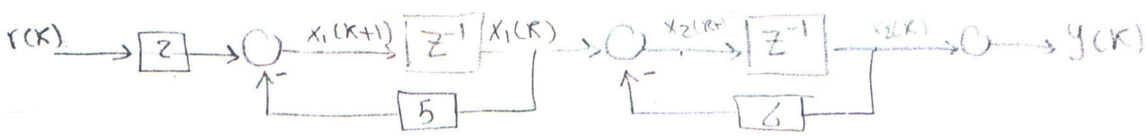
$$y(k) = \begin{pmatrix} 2 & 0 \end{pmatrix} X(k)$$



2) observable form

$$X(k+1) = \begin{pmatrix} 0 & -5 \\ 1 & -6 \end{pmatrix} X(k) + \begin{pmatrix} 2 \\ 0 \end{pmatrix} r(k)$$

$$y(k) = \begin{pmatrix} 0 & 1 \end{pmatrix} X(k)$$



(b) $y(k+2) + 6y(k+1) + 5y(k) = 3r(k+2) + r(k+1) + 2r(k)$

$$z^2 Y(z) + 6zY(z) + 5Y(z) = 3z^2 R(z) + zR(z) + 2R(z)$$

$$Y(z)\left[z^2 + 6z + 5\right] = R(z)\left[3z^2 + z + 2\right]$$

$$\frac{Y(z)}{R(z)} = \frac{3z^2 + z + 2}{z^2 + 6z + 5}$$

2-15

$$\frac{\overset{3}{B_0} z^2 + \overset{1}{B_1} z + \overset{2}{B_2}}{z^2 + \underset{6}{\alpha_1} z + \underset{5}{\alpha_2}}$$

controlable form

$$X(k+1) = \begin{pmatrix} 0 & 1 \\ -5 & -6 \end{pmatrix} X(k) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r(k)$$

$$y(k) = (-13 \quad -17)\, X(k) + 3\, r(k)$$

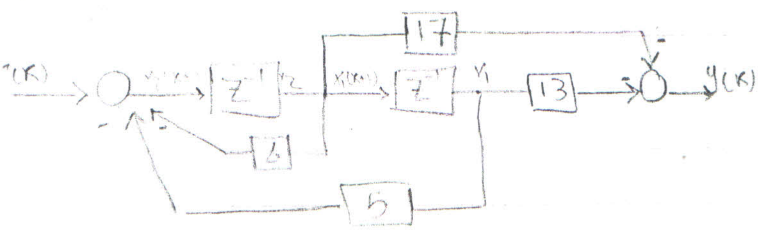
$y(k) = (A_1 \quad A_2) + B_0$

$A_1 = B_2 - B_0 \alpha_2$

$A_2 = B_1 - B_0 \alpha_1$

$A_1 = 2 - 3*5 = 2 - 15 = -13$

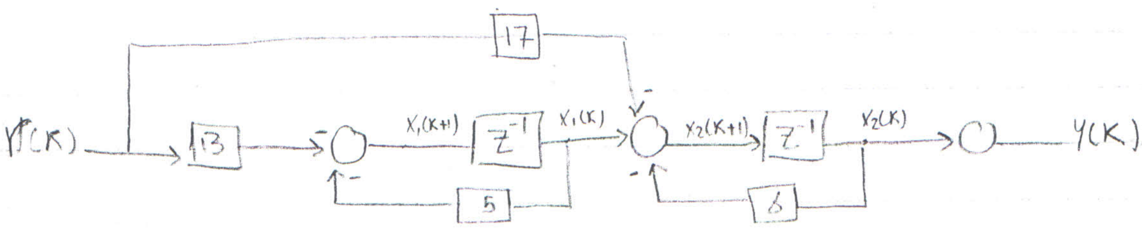
$A_2 = 1 - 3*6 = 1 - 18 = -17$



2) observable form

$$X(k+1) = \begin{pmatrix} 0 & -5 \\ 1 & -6 \end{pmatrix} X(k) + \begin{pmatrix} -13 \\ -17 \end{pmatrix} r(k)$$

$$y(k) = (0 \quad 1)\, X(k) + 3\, r(k)$$



sheet (6)(4)

(C) $\dfrac{Y(Z)}{R(Z)} = \dfrac{Z^2+2}{Z^3+3Z^2-4Z+2}$

$$\frac{Y(Z)}{V(Z)} \cdot \frac{V(Z)}{R(Z)} = Z^2+2 \cdot \frac{1}{Z^3+3Z^2-4Z+2}$$

1) controlable form

$\rightarrow Y(Z) = Z^2 V(Z) + 2V(Z)$

$y(K) = V(K+2) + 2V(K)$

$y(K) = X_3(K) + 2X_1(K)$

$\rightarrow R(Z) = Z^3 V(Z) + 3Z^2 V(Z) - 4Z V(Z) + 2V(Z)$

$R(K) = V(K+3) + 3V(K+2) - 4V(K+1) + 2V(K)$

$R(K) = X_3(K+1) + 3X_3(K) - 4X_2(K) + 2X_1(K)$

ASSum

$V(K) = X_1(K)$ $\qquad$ $X_1(K+1) = X_2(K)$

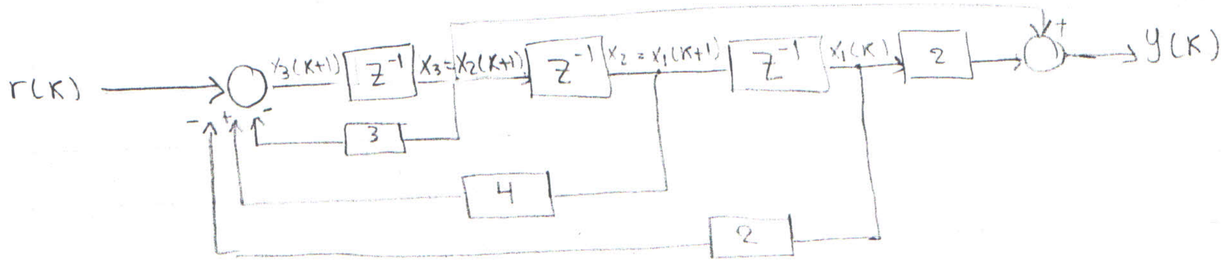
$V(K+1) = X_2(K)$ $\qquad$ $X_2(K+1) = X_3(K)$

$V(K+2) = X_3(K)$ $\qquad$ $X_3(K+1) = R(K) - 3X_3(K) + 4X_2(K) - 2X_1(K)$

$\cancel{V(K+3) = X_4(K)}$ $\qquad$ $\cancel{X_4(K+1) =}$

$$X(K+1) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 4 & -3 \end{pmatrix} \begin{pmatrix} X_1(K) \\ X_2(K) \\ X_3(K) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} r(K)$$

$$y(K) = \begin{pmatrix} 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_1(K) \\ X_2(K) \\ X_3(K) \end{pmatrix} + D\,\cancel{r(K)}$$



$$\begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 4 & -3 \end{matrix}$$

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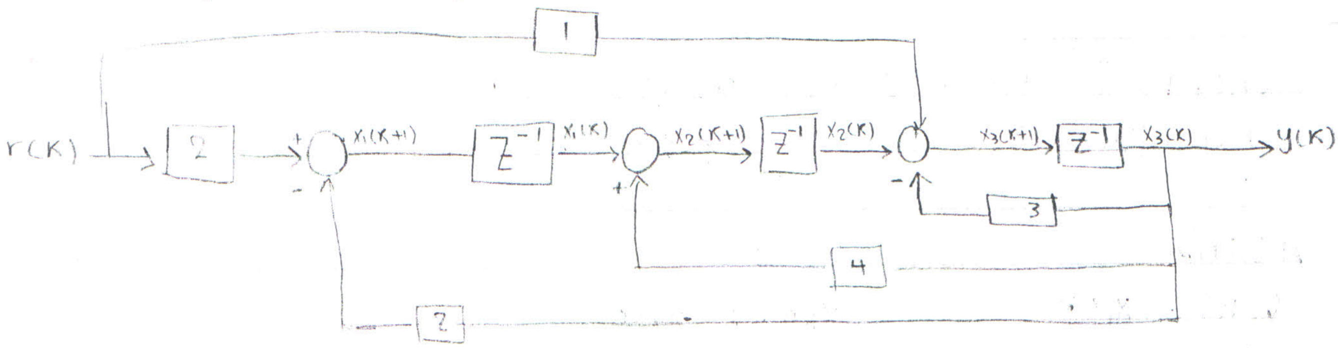
$(2 \quad 0 \quad 1)$

2) observable form

$$X(K+1) = \begin{pmatrix} 0 & 0 & -2 \\ 1 & 0 & 4 \\ 0 & 1 & -3 \end{pmatrix} \begin{pmatrix} x_1(K) \\ x_2(K) \\ x_3(K) \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} r(K)$$

$$y(K) = (0 \quad 0 \quad 1)\, X(K)$$

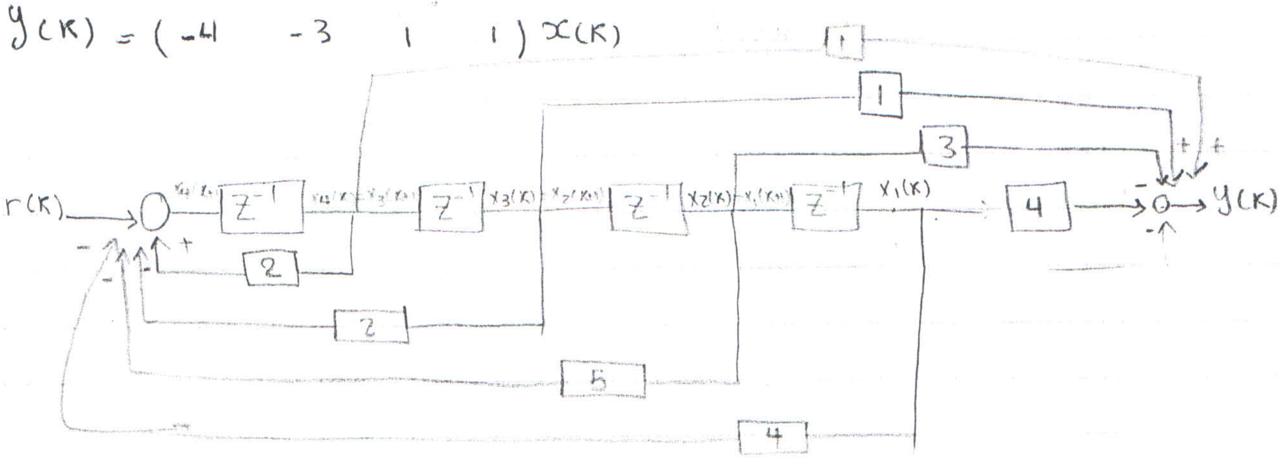


(d) $$\frac{Y(Z)}{R(Z)} = \frac{Z^3 + Z^2 - 3Z - 4}{Z^4 - 2Z^3 + 2Z^2 + 5Z + 4}$$

1- Controlable form

$$X(K+1) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & -5 & -2 & 2 \end{pmatrix} \begin{pmatrix} x_1(K) \\ x_2(K) \\ x_3(K) \\ x_4(K) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} r(K)$$

$$y(K) = (-4 \quad -3 \quad 1 \quad 1)\, x(K)$$

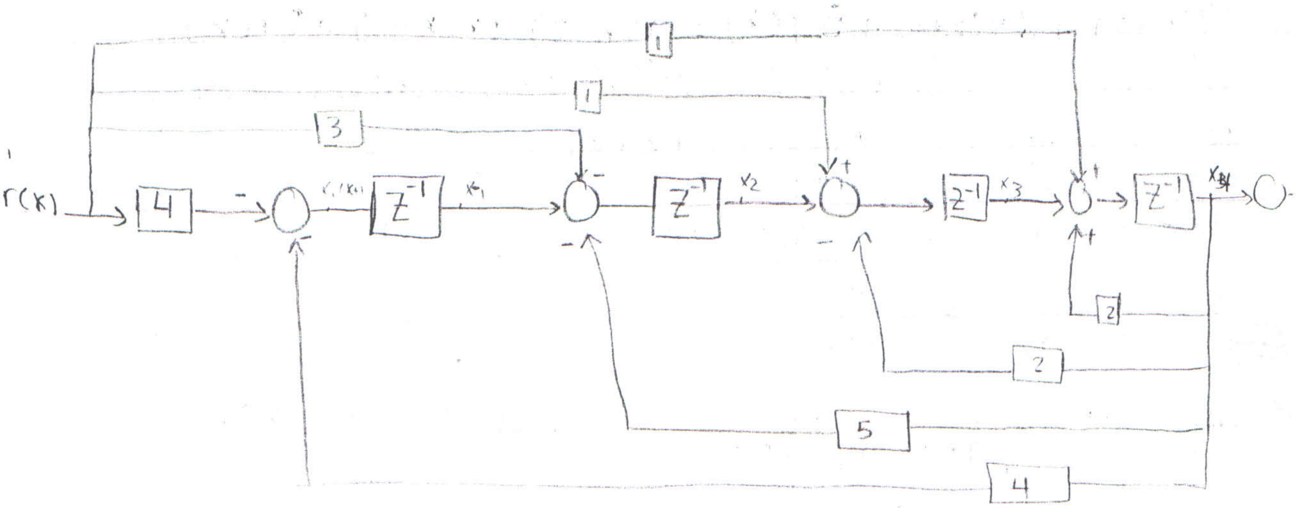


$$\begin{matrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

2- observable form sheet(6) (5)

$$x(K+1)=\begin{pmatrix} 0 & 0 & 0 & -4 \\ 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{pmatrix} x(K) + \begin{pmatrix} -4 \\ -3 \\ 1 \\ 1 \end{pmatrix} r(K)$$

$$y(K) = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix} x(K)$$



3 obtain the Diagonal State Space form & Draw the State Diagram

(a) $y(K+2)+6y(K+1)+5y(K)=2r(K)$

$$z^2 Y(z)+6zY(z)+5Y(z)=2R(z)$$

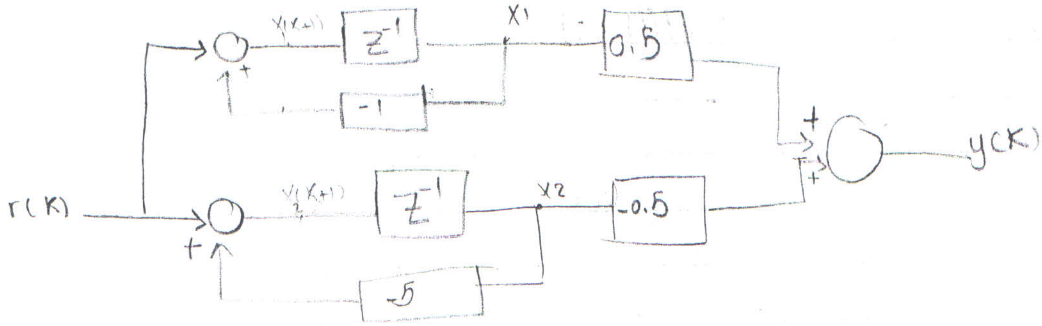
$$Y(z)\left[z^2+6z+5\right]=2R(z)$$

$$\frac{Y(z)}{R(z)}=\frac{2}{z^2+6z+5}=2\left[\frac{\overset{0.25}{A}}{(z+1)}+\frac{\overset{-0.25}{B}}{(z+5)}\right]=2\left[\frac{1}{(z+1)(z+5)}\right]$$

$$=2\left[\frac{0.25}{z+1}-\frac{0.25}{z+5}\right]=\frac{0.5}{z+1}-\frac{0.5}{z+5}$$

$$x(K+1)=\begin{pmatrix} -1 & 0 \\ 0 & -5 \end{pmatrix}\begin{pmatrix} x_1(K) \\ x_2(K) \end{pmatrix}+\begin{pmatrix} 1 \\ 1 \end{pmatrix} r(K)$$

$$y(K)=\begin{pmatrix} 0.5 & -0.5 \end{pmatrix} x(K)$$



(b) $y(K+2) + 6y(K+1) + 5y(K) = 3r(K+2) + r(K+1) + 2r(K)$

$z^2 Y(z) + 6zY(z) + 5Y(z) = \cancel{3R^2}\, 3z^2 R(z) + zR(z) + 2R(z)$

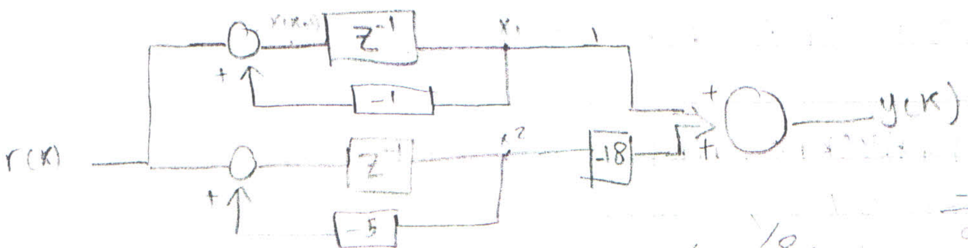
$Y(z)[z^2+6z+5] = R(z)[3z^2+z+2]$

$\frac{Y(z)}{R(z)} = \frac{3z^2+z+2}{z^2+6z+5} = \frac{3z^2+z+2}{(z+1)(z+5)} = \frac{A}{z+1} + \frac{B}{z+5}$ $\quad A \to 1, \quad B \to -18$

$\frac{75-5+2}{-4}$

$\frac{Y(z)}{R(z)} = \frac{1}{z+1} + \frac{-18}{z+5}$

$x(K+1) = \begin{pmatrix} -1 & 0 \\ 0 & -5 \end{pmatrix} x(K) + \begin{pmatrix} 1 \\ 1 \end{pmatrix} r(K) \qquad y(K) = (1 \quad -18)\, x(K)$



(c) $\frac{Y(z)}{R(z)} = \frac{z-0.5}{z(z+0.5)(z+0.25)} = \frac{A}{z} + \frac{B}{z+0.5} + \frac{C}{z+0.25}$ $\quad A \to 0.66, \quad B \to 8,\ 0.125, \quad C \to 12$

$\frac{-0.5}{0.5+0.25} \qquad \frac{-1}{-0.5+0.25} \qquad \frac{-0.75}{-0.25(0.25)}$

$x(K+1) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -0.25 \end{pmatrix} x(K) + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} r(K)$

$y(K) = (0.66 \quad 0.125 \quad 12)\, x(K)$

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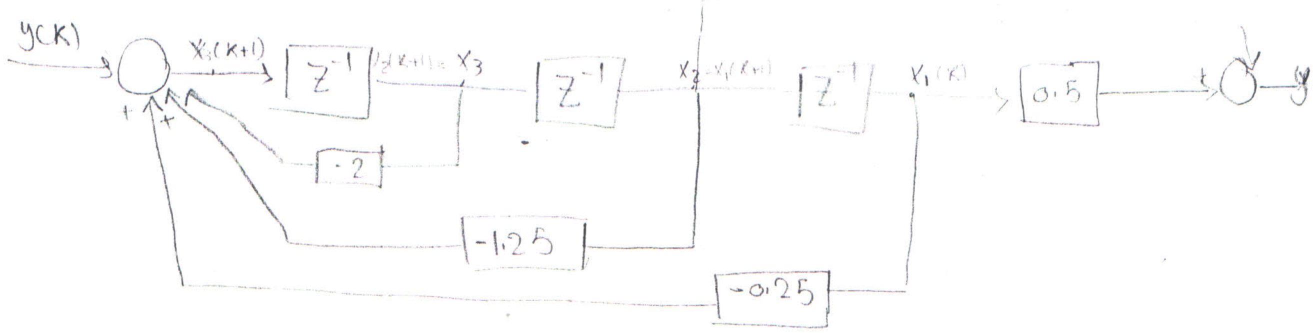
$$\frac{Y(Z)}{R(Z)} = \frac{Z-0.5}{(Z+0.5)^2(Z+1)} = \frac{Z-0.5}{Z^3+2Z^2+1.25Z+0.25}$$

1. controlable

$$X(K+1) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.25 & -1.25 & -2 \end{pmatrix} X(K) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} r(K)$$

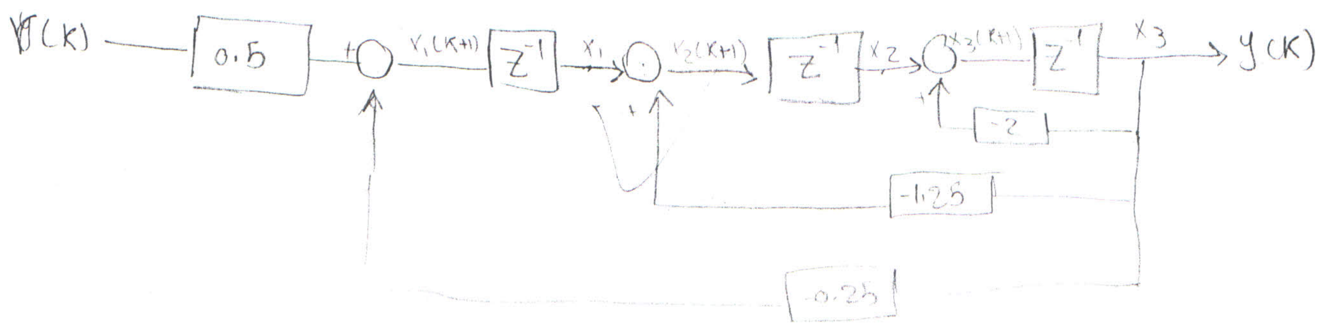
$$y(K) = \begin{pmatrix} 0.5 & 1 & 0 \end{pmatrix} X(K)$$



2. observable

$$X(K+1) = \begin{pmatrix} 0 & 0 & -0.25 \\ 1 & 0 & -1.25 \\ 0 & 1 & -2 \end{pmatrix} X(K) + \begin{pmatrix} 0.5 \\ 1 \\ 0 \end{pmatrix} r(K)$$

$$y(K) = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} X(K)$$



3. Diagonal

$$= \frac{A\cancel{+B}}{(Z+0.5)^2} + \frac{B}{Z+0.5} + \frac{C}{Z+1}$$

$$X(K+1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -1 \end{pmatrix} X(K) + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} r(K)$$

$y(K) = (\underset{-6}{A} \quad \underset{6}{B} \quad \underset{-2}{C}) \, x(K)$